

C. U. SHAH UNIVERSITY

Winter Examination-2021

Subject Name: Advanced Calculus

Subject Code: 4SC03ADC1

Branch: B.Sc. (Mathematics)

Semester: 3

Date: 14/12/2021

Time: 02:30 To 05:30

Marks: 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

- Q-1 Attempt the following questions: (14)**
- a) If $f(x, y) = (3x)(2y) + 4 \cos(xy)$ then $f_{xx}(0,1) = \underline{\hspace{2cm}}$. 01
 (a) 0 (b) -1 (c) -4 (d) None
- b) If $u(x, y) = \frac{x^3+y^3}{x-y}$ then the value of $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \underline{\hspace{2cm}}$. 01
 (a) 0 (b) u (c) $2u$ (d) None
- c) If $f(x, y) = x^2 + 2xy + y^2$ then which of the following is true? 01
 (a) f is not homogeneous function
 (b) f is not homogeneous function with degree one
 (c) f is homogeneous function with degree two
 (d) All of above.
- d) If $f(x, y) = x^2 + y^2 + 6x + 12$ then extreme points of f is _____. 01
 (a) $(-3,0)$ (b) $(3,0)$ (c) $(0,0)$ (d) None
- e) If $f(x, y) = 0$ is implicit function then $\frac{dy}{dx} = \underline{\hspace{2cm}}$. 01
 (a) $\frac{f_x}{f_y}$ (b) $-\frac{f_x}{f_y}$ (c) $\frac{f_y}{f_x}$ (d) $-\frac{f_y}{f_x}$
- f) The relation between Beta and Gamma function is given by _____. 01
 (a) $\frac{\Gamma m \Gamma n}{\Gamma(m+n)}$ (b) $\frac{\Gamma m \Gamma n}{\Gamma(m-n)}$ (c) $\frac{\Gamma m \Gamma n}{\Gamma(m+n+1)}$ (d) None
- g) The value of $\Gamma\left(\frac{1}{2}\right) = \underline{\hspace{2cm}}$. 01
 (a) π (b) $\sqrt{\pi}$ (c) $\frac{\pi}{2}$ (d) None
- h) Find $\Gamma(5,7)$. 01
- i) True/False: If u and v are function of x and y then $JJ' = 0$. 01
- j) Define: Concave Downward 01
- k) Evaluate: $B\left(\frac{1}{4}, \frac{3}{4}\right)$ 02
- l) If $z = f(u, v)$, $u = g(x, y)$ and $v = h(x, y)$ then $\frac{\partial z}{\partial y} = \underline{\hspace{2cm}}$. 02



Attempt any four questions from the Q-2 to Q-8

Q-2 Attempt all questions (14)

(a) Investigate the continuity of function $f(x, y)$ at origin. **05**

$$f(x, y) = \begin{cases} \frac{xy}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

(b) If $z(x + y) = x^2 + y^2$ then prove that $\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)^2 = 4\left(1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)$ **05**

(c) If $u = x^y$ show that $\frac{\partial^3 u}{\partial x^2 \partial y} = \frac{\partial^3 u}{\partial x \partial y \partial x}$. **04**

Q-3 Attempt all questions (14)

(a) State and prove Euler's theorem for homogeneous function. **05**

(b) If $u = f(e^{y-z}, e^{z-x}, e^{x-y})$ then show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$. **05**

(c) If $\sin(xy) - e^{xy} - x^2y = 0$ then find $\frac{dy}{dx}$. **04**

Q-4 Attempt all questions (14)

(a) If $u = \tan^{-1}(x^2 + y^2)$ then prove that $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 2 \sin u \cos 3u$. **07**

(b) If $x = e^u \sec v$ and $y = e^u \tan v$ then prove that $J \cdot J' = 1$ **07**

Q-5 Attempt all questions (14)

(a) State and prove Taylor's series for function of two variables. **07**

(b) Verify Euler's theorem for $u = (ax + by)^{\frac{1}{3}}$. **05**

(c) State duplication formula. **02**

Q-6 Attempt all questions (14)

(a) Find concave upward and concave downward for $f(x) = (x^2 + 4x + 5)e^{-x}$ also find point of inflection. **05**

(b) Find the maximum value of $v(x, y, z) = xyz$ subject to the constraint $2x + 2y + z = 108$. **05**

(c) If $u = x + y$ and $v = \frac{x}{x+y}$ then find $\frac{\partial(u,v)}{\partial(x,y)}$. **04**

Q-7 Attempt all questions (14)

(a) Evaluate: $\int_0^\infty \frac{x^2}{(1+x^4)^3} dx$ **05**

(b) Evaluate: $\int_3^7 \sqrt[4]{(x-3)(7-x)} dx$ **05**

(c) Prove that $\Gamma(n) = 2 \int_0^\infty e^{-x^2} x^{2n-1} dx$ **04**

Q-8 Attempt all questions (14)

(a) Find the maximum and minimum value of $x^2y^2 - 5x^2 - 8xy - 5y^2$. **07**



(b) Expand e^{xy} in power of $(x - 1)$ and $(y - 1)$ using Taylor's expansion. **05**

(c) Evaluate: $\lim_{(x,y) \rightarrow (1,2)} \frac{2x^2y}{x^2+y^2+1}$ **02**

